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EXERCISES.

93

FIND the radius of the circle inscribed in the evolute of an ellipse and the ratio in which each point of contact divides the quadrant on which it lies.

[*R. H. Graves.*]

94

O is the centre of the circumscribed circle of ABC , and D, E, F the middle points of its sides. Show that

$$OD^2 + OE^2 + OF^2 = 2R'(2R' - r'),$$

where R', r' are the radii of the circumscribed and inscribed circles of the triangle of the feet of the altitudes.

[*R. D. Bohannan.*]

95

IN exercise 65 let k, l be the lengths of the perpendiculars CD, CE drawn at right angles to CA, CB to meet the base in D, E . Show that

$$\frac{m}{c} = \frac{k}{a} \cdot \frac{l}{b}. \quad [\textit{Ormond Stone.}]$$

96

GIVEN on the ground a circular curve of known radius intersecting a given straight line at a given point, and inclined to it at that point at a given angle; it is required to determine the radius of a second circular arc which shall be tangent both to the given curve and to the given line at another given point.

[*Calvin Whiteley.*]

97

IN the triangle ABC two lines drawn from C trisect the side AB . Given c, C , and the angle ϕ between the trisecants; to solve the triangle.

[*Marcus Baker.*]

98

THE eccentric anomalies of three points on an ellipse are ϕ_1, ϕ_2, ϕ_3 . Show that the area of their triangle is

$$\Delta = 2ab \sin \frac{\phi_2 - \phi_3}{2} \sin \frac{\phi_3 - \phi_1}{2} \sin \frac{\phi_1 - \phi_2}{2};$$

the centre of its circumscribing circle

$$x = + \frac{c^2}{a} \cos \frac{\phi_2 + \phi_3}{2} \cos \frac{\phi_3 + \phi_1}{2} \cos \frac{\phi_1 + \phi_2}{2},$$

$$y = - \frac{c^2}{b} \sin \frac{\phi_2 + \phi_3}{2} \sin \frac{\phi_3 + \phi_1}{2} \sin \frac{\phi_1 + \phi_2}{2};$$

and hence show that the centre of curvature of the ellipse at (x, y) is

$$X = +\frac{c^2x^3}{a^4}, \quad Y = -\frac{c^2y^3}{b^4}. \quad [Wm. M. Thornton.]$$

99

FROM a full cask of wine a quantity is taken out at random and the cask filled with water, and then a quantity of the mixture is taken out at random and the cask again filled with water. What is the probability that the cask now contains more wine than water? [Artemas Martin.]

SELECTED.

100

THE points O, O' defined by the equations in trilinears

$$aa : b\beta : c\gamma = c^2a^2 : a^2b^2 : b^2c^2,$$

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are called the Brocard points. The angles $OCA, OAB, OBC, O'BA, O'CB, O'AC$ are called the Brocard angles.

1. Show that the Brocard angles are all equal each to

$$\cot^{-1}[\cot A + \cot B + \cot C].$$

2. Find the equation to the Brocard line OO' .

3. Find the equation to the Brocard circle through O, O' and the centre of the circumscribed circle.

4. Given the base BC and the Brocard angle of a triangle, find the locus of the vertex.

5. Show that the medians bisect the angles between the bisectrices of the angles of ABC and the "symmedian lines"

$$\frac{a}{a} = \frac{\beta}{b} = \frac{\gamma}{c}.$$

6. Show that the Brocard circle contains the symmedian point.

101

EXPRESS in terms of $\sin^{-1}x$ and $\sin^{-1}y$

$$\tan^{-1} \frac{x+y}{\sqrt{(1-x^2)} + \sqrt{(1-y^2)}}.$$

102

FIND the relation connecting x, y, z when

$$\cot^{-1}(x+y+z-xyz) = \cot^{-1}x + \cot^{-1}y + \cot^{-1}z.$$

103

INTO a conical wine glass a spherical ball is dropped. Find the ratio of the concealed surfaces of the ball and the inside of the glass.